Raman and non Raman electromagnetically induced transparency resonances in a degenerate four level system

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We study coherence effects in an atom having three metastable levels and single life-time broadened upper state. Two laser fields couple ground states to an excited one and an radio-frequency (rf) field operates between ground state levels. We consider the case when the common level for the rf-field and one of the laser field (to which we refer as the probing field) transitions is twice degenerate. We predict a novel type of dark states (we call them non Raman dark states), which in contrary to Raman resonances, do not require the fulfillment of the two-photon resonance condition. We realize the configuration in the atomic hydrogen and show that non Raman resonances are determined by the geometry of the rf and probing field's polarizations. For ultra-cold atoms non Raman resonances arise when the polarization vectors of the fields are parallel or perpendicular to each other. For an atomic gas at room temperatures electromagnetically induced transparency (EIT) resonances associated with Raman and non Raman dark states are studied by taking into account spin-exchange relaxation in the ground state of the hydrogen atom. We establish properties of non Raman dark states by investigating the relation of EIT resonances to the polarization geometry.

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I. INTRODUCTION

Coherent population trapping (CPT) is one of the remarkable phenomenon associated with quantum coherence and interference in a three-level atom. The essence of coherent population trapping is that, under certain conditions, there exists a coherent superposition among atomic state (the dark state), which is decoupled from the coherent and dissipative interactions [1]. When an atomic system is prepared in the dark state electromagnetically induced transparency (EIT) can be observed where the atomic coherence cancels or reduces absorptions [2, 3]. The importance of EIT resonance stems from the fact that it gives rise to greatly enhanced susceptibility (the real part of the susceptibility) in the spectral region of induced transparency of the medium which leads to the enhancement of the refractive index [4, 5]. Another interesting nonlinear effect related to dark states is the adiabatic population transfer, which allows to prepare the atom in a desired state and to control transitions between atomic levels by means of adiabatically changing continuous radiation waves [6]. The interest to dark states is due not only to the fascinating physics involving quantum interference but also to the fact that there are many potential applications such as lasing without inversion [7–9], quantum information processing and storage [10], subrecoil laser cooling [11] and atom interferometry [12].

The dark state was observed for the first time by G. Alzetta et al. [13] in the Lambda (Λ) configuration in which two ground states are coupled to a single excited state by a bichromatic field. The single dark state appears when two laser fields satisfy the two-photon Raman resonance condition for the transition between two ground states. It is possible to enlarge the domain of the dark state based physics by adding an additional metastable level to the Lambda system. In this case double dark state structure is observed in the configuration in which the additional level is coupled by a rf field to one of the Lambda system's ground state [14–17]. The appearence of double dark state structure is associated with the Autler-Townes splitting of atomic levels induced by the rf field. The resonances associated with the double dark states can be made absorptive or transparent and their optical properties such as width and position can be manipulated by adjusting the coherent interaction [18]. In the configuration where three laser field operates between ground states and excited one, two coupling laser fields can be used for slowing down one weak probing field. As a result two dark states appear in the system which lead to two EIT resonances and two group velocities for the probe field [19]. Along with the Lambda system it is also of great interest atomic coherent states and associated nonlinear optical effects in a so-called Vee (V) and Cascade (Ξ) configurations where more than one level may be unstable. In recent years, many extensive studies have been devoted to four-level systems which are the generalization of these three-level configurations [20–24].

This work is adressed to the study of coherence effects in the four level system which one metastable level is twice degenerate (the configuration is shown in Fig. 1(a)). The rf field induced dynamical Stark effect splits the degenerate level into three sub-levels. As a result three dark states appear when the coupling and the probing fields satisfy the two-photon resonance conditions between corresponding lower state's transitions (see Fig. 1(b)). This triple dark state structure is the straightforward generalization of the double dark state structure observed in non degenerate four level systems (see references above). Along with Raman dark states the system admits another type of dark states which have no ana-

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logues in non-degenerate four level systems. For certain polarizations of the rf and probing fields qualitatively new dark states occur which do not require the fulfillment of the two-photon resonance condition. In the following we will refer to them as non Raman dark states to distinguish them from Raman ones which occur when the two-photon resonance condition takes place. The origin of non Raman resonances is related to the degeneracy of the configuration: The rf field and probing field polarization degrees of freedom form entangled states in the two dimensional quantum space of the degenerate level, and, as a consequence interference effects occur in the system resulting in non Raman resonances. The purpose of the present work is to study properties of non Raman resonances. We anticipate that non Raman dark states together with multiple Raman ones can enlarge the domain of the dark state based physics. We prefer in the theoretical treatments to use a simple physical model that can account for the major characteristics of the configuration of Fig. 1(a). For this reason we realize the degenerate four level system in the atomic hydrogen. We consider the ground state relaxation due to spin-exchange collisions which is sufficiently good approximation for a dense hydrogen gas at room temperatures. Using the full set of density matrix equations we establish properties of EIT resonances associated with Raman and non-Raman dark states.

The paper is organized as follows. In the following section we setup the model and study its dark states using the dressed picture formalism. In Sec. III we establish main properties of Raman and non Raman resonances in a dense hydrogen gas at room temperatures. Our conclusion follows in Sec. IV. Density matrix equations for the system are given in the Appendix.

II. DARK STATES

In this work we consider a degenerate four level atom with triplet ground levels E_1, E_2, and E_3 and an excited level E_4 interacting with two lasers and one rf field as shown in Fig. 1. A twice degenerate level E_1 is spanned by the pair of orthogonal vectors $\mid 1>$ and $\mid 1'>$ and the wave functions of the remaining three levels are $\mid 2>$, $\mid 3>$ and $\mid 4>$. A laser of frequency ω_c is resonant with the $E_2 \rightarrow E_4$ transition and is referred to as the coupling field. A rf field of frequency ω_r is resonant with the $E_1 \rightarrow E_3$ transition and the absorption spectrum is obtained by scanning the frequency ω_p of the second laser, referred to as the probing field which operates quasi-resonantly between the E_1 and E_4 levels.

The scanning parameter which we use in the article is the probing field frequency detuning from the resonance

$$\triangle = \frac{E_4 - E_1}{\hbar} - \omega_p. \tag{1}$$

The Rabi frequencies of the coupling and probing fields are denoted by Ω_c and Ω_p, Ω'_p, respectively. The presence

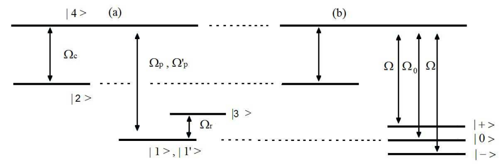

FIG. 1. (1a) The degenerate four level system: three metastable and single life-time broadened levels are driven by the coupling (Ω_c), probing (Ω_p) and radio frequency (Ω_r) fields. The lowest level is twice degenerate. (2a) The degenerate four level system in the dressed picture. Three Raman dark states occur when the coupling and probing fields satisfy the two-photon resonance condition. Non Raman dark states arise when one of the dressed Rabi frequencies Ω or Ω_0 is zero.

of the two Rabi frequencies for the probing light is associated with the degeneracy of the level E_1. Not primed and primed Rabi frequencies are proportional to electric dipole transitions between the ground states $\mid 1>$ and $\mid 1'>$ and the excited state $\mid 4>$. For the same reason there are two Rabi frequencies for the rf wave which we denote by Ω_r and Ω'_r. They are proportional to magnetic dipole transitions within hyperfine levels. The semiclassical Hamiltonian of the quantum system in the rotating wave frame is

$$\begin{aligned} H = & \triangle(\mid 2><2\mid + \mid 4><4\mid) \\ & - \frac{1}{2}\left(\Omega_r \mid 3><1\mid + \Omega'_r \mid 3><1'\mid + H.c.\right) \\ & - \frac{1}{2}\left(\Omega_c \mid 4><2\mid + \Omega_c \mid 2><4\mid\right) \\ & - \frac{1}{2}\left(\Omega_p \mid 4><1\mid + \Omega'_p \mid 4><1'\mid + H.c.\right), \end{aligned} \tag{2}$$

where $H.c.$ stands for the hermitian conjugation. When treating a strong photon-atom interaction, it is often convenient to introduce the dressed state picture, which allows us to gain more physical insight [25]. In a dressed state picture the E_1 and E_3 levels and the rf field is treated as a coupled '"atom + rf field" system which Hamiltonian is given by the expression in the second line of (2). Eigenvectors

$$\mid -> = \frac{1}{2\sqrt{2}}\left(\frac{\overline{\Omega}_r}{\Sigma}\mid 1> + \frac{\overline{\Omega}'_r}{\Sigma}\mid 1'> + \mid 3>\right), \tag{3a}$$

$$\mid 0> = \frac{\Omega'_r}{2\Sigma}\mid 1> - \frac{\Omega_r}{2\Sigma}\mid 1'>, \tag{3b}$$

$$\mid +> = \frac{1}{2\sqrt{2}}\left(\frac{\overline{\Omega}_r}{\Sigma}\mid 1> + \frac{\overline{\Omega}'_r}{\Sigma}\mid 1'> - \mid 3>\right) \tag{3c}$$

of the "atom + rf field" Hamiltonian form a ladder of triplet as shown in Fig. 1 (b) where the energy difference is defined by the rf field induced light shift

$$\Sigma = \frac{1}{2}\sqrt{\mid \Omega_r \mid^2 + \mid \Omega'_r \mid^2}. \tag{4}$$

In a similar fashion we may treat the E_4 and E_1 levels coupled by the the probing field as the "atom + probing

field" system which Hamiltonian is given by the expression in the fourth line of (2). In the dressed state picture this Hamiltonian reads

$$-\frac{1}{2}\Omega_0 \mid 4 >< 0 \mid -\frac{1}{2}\Omega \mid 4 > \left(\frac{< + \mid + < - \mid}{\sqrt{2}}\right) + H.c., \tag{5}$$

where

$$\Omega_0 = \frac{\Omega_p \Omega_r' - \Omega_p' \Omega_r}{\Sigma}, \quad \Omega = \frac{\Omega_p \overline{\Omega}_r + \Omega_p' \overline{\Omega}'}{\Sigma}. \tag{6}$$

are dressed Rabi frequencies. In the absence of relaxations within metastable states (the case of ultra cold atomic gas) dark states of the quantum system may be described by means of the Schrödinger equation. A quantum superposition state is decoupled from the coherent interactions if it is orthogonal to the excited state. Under this condition the Hamiltonian (2) admits three eigenfunctions provided that the frequency detuning is zero or equal to $\pm\Sigma$. Since the coupling field is in exact resonance with the transition $\mid 2 > \to \mid 4 >$ the condition $\triangle = 0$ is equivalent to the two-photon resonance condition for the $\mid 2 > \to \mid 0 >$ transition. Other two Raman dark states occur when the two-photon resonance conditions for the $\mid 2 > \to \mid \pm >$ transitions are valid.

We have qualitatively new dark states when one of the Rabi frequencies in (6) is zero. When $\Omega_0 = 0$ the state $\mid 0 >$ is the eigenfunction of the Hamiltonian. This state exists for an arbitrary value of the frequency detuning. Consequently no two-photon resonance condition is required. In a similar fashion when $\Omega = 0$ we have two non Raman dark states $\mid \pm >$. To understand the origin of non Raman dark states we realize the degenerate four level system in the hydrogen atom using hyperfine sublevels of the ground states. The hyperfine level of the ground state $S_{\frac{1}{2}}$ with the total angular momentum F is spanned by wave functions $\mid S, F, m >$ where the magnetic quantum number m enumerates Zeeman sub-levels of the F-hyperfine level. We make the following choice

$$\mid 1 > = \mid S, 1; 1 >, \tag{7a}$$
$$\mid 1' > = \mid S, 1; -1 >, \tag{7b}$$
$$\mid 2 > = \mid S, 1; 0 >, \tag{7c}$$
$$\mid 3 > = \mid S, 0; 0 > . \tag{7d}$$

As the excited state we choose the Zeeman sublevel of the hyperfine level with the total angular momentum $F = 0$ in the $P_{\frac{1}{2}}$ fine structure

$$\mid 4 > = \mid P, 0; 0 > . \tag{8}$$

Transitions from the ground state into the excited state are described by the electric dipole Hamiltonian $-\vec{d}\vec{\mathcal{E}}(t)$, where $\vec{d}$ is the electric dipole operator of the atom and

$$\vec{\mathcal{E}}(t) = \frac{1}{2}\vec{\mathcal{E}} e^{-i\omega_p t} + \frac{1}{2}\vec{\mathcal{E}}^* e^{i\omega_p t} \tag{9}$$

is the electric field. The Rabi frequencies

$$\Omega_p = \frac{1}{\hbar} < P, 0, 0 \mid \vec{d}\vec{\mathcal{E}} \mid S, 1; 1 >, \tag{10a}$$
$$\Omega_p' = \frac{1}{\hbar} < P, 0, 0 \mid \vec{d}\vec{\mathcal{E}} \mid S, 1; -1 > \tag{10b}$$

take the form

$$\Omega_p = \frac{d}{\hbar}\frac{\mathcal{E}_x + i\mathcal{E}_y}{\sqrt{2}}, \quad \Omega_p' = \frac{d}{\hbar}\frac{\mathcal{E}_x - i\mathcal{E}_y}{\sqrt{2}}. \tag{11}$$

$\mathcal{E}_x$ and $\mathcal{E}_y$ are x and y components of the electric field and d is the atomic dipole moment. Levels within the same hyperfine structure have the same parity and the rf-field couples E_1 and E_3 levels by the magnetic dipole Hamiltonian $-\vec{m}\vec{\mathcal{H}}(t)$, where $\vec{m}$ is the magnetic dipole operator of the atom and $\vec{\mathcal{H}}(t)$ is the magnetic component of the rf field. Repeating for the rf field the above derivations we obtain

$$\Omega_r = \frac{m}{\hbar}\frac{\mathcal{H}_x + i\mathcal{H}_y}{\sqrt{2}}, \quad \Omega_r' = \frac{m}{\hbar}\frac{\mathcal{H}_x - i\mathcal{H}_y}{\sqrt{2}}. \tag{12}$$

Here m is the magnetic moment which is proportional to the Bohr magneton. Inserting (11) and (12) in (6) we finally obtain

$$\Omega_0 = \frac{id}{\hbar}\frac{\mathcal{E}_y\mathcal{H}_x - \mathcal{E}_x\mathcal{H}_y}{\mid \vec{\mathcal{H}} \mid}, \quad \Omega = \frac{d}{\hbar}\frac{\mathcal{E}_x\overline{\mathcal{H}}_x + \mathcal{E}_y\overline{\mathcal{H}}_y}{\mid \vec{\mathcal{H}} \mid}. \tag{13}$$

The dressed Rabi frequencies are defined by the scalar product and by the z component of the vector product between the polarization vectors of the rf and probing fields. In the case of linear polarizations non Raman dark states occur when the probing field polarization is parallel or perpendicular to that of the rf field.

III. PROPAGATION OF WEAK PROBE FIELD

To quantify the properties of EIT resonances associated with Raman and non Raman dark states for an atomic gas at room temperatures we examine the linear response of the system (Fig.1 a) using the full set of density matrix equations given in the Appendix A. In a sufficiently dense hydrogen gas ground state relaxations are limited by spin-exchange mechanism [27, 28]. The evolution of the atom due to the spin-exchange scattering is described by equations (A5). We will use linear approximation for the spin-exchange relaxation which is valid when the average spin of the gas is close to zero [29]. The spin- exchange rate γ is a function of the density and the temperature of a gas. In this paper a ratio of $\gamma/\Gamma = 10^{-4}$ is used. We assume that the rf field is linear polarized along the x direction ($\mathcal{H}_y = 0$) and the probing electric field is linear polarized in the $x0y$ plane

$$\mathcal{E}_x = \mathcal{E}\cos\psi, \quad \mathcal{E}_y = \mathcal{E}\sin\psi, \tag{14}$$

where ψ is the angle between $\vec{\mathcal{E}}$ and $\vec{\mathcal{H}}$.

The linear response σ of atoms to a weak probing field defines the dielectric polarization of the medium

$$\mathcal{P}_x = 2d\varrho(\sigma_{41'} + \sigma_{41}), \quad \mathcal{P}_y = 2id\varrho(\sigma_{41'} - \sigma_{41}), \tag{15}$$

where ϱ is the number of atoms per unit volume in the interaction region. Using solutions of the Liouville equation (A1) we find that non diagonal components of the dielectric susceptibility tensor are zero. As a result we have

$$\mathcal{P}_x = \chi_x \mathcal{E}_x, \qquad \mathcal{P}_y = \chi_y \mathcal{E}_y. \tag{16}$$

When the rf field is switched off we have $\chi_x = \chi_y$ and the medium becomes isotropic for the probe field. The E_3 level is decoupled from other ones and we arrive at the Lambda system with single EIT resonance which appears at the zero frequency detuning as shown in the Fig. 2(a).

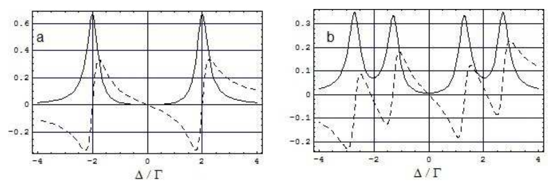

FIG. 2. Imaginary (solid lines) and real (dashed lines) parts of the susceptibility function $\chi(\psi)$ in the units of λ for the degenerate four level system. (a) the case when the rf field is switched off. The same is true for nonzero rf field when the polarization vector of the rf field is parallel to that of the probing field. (b) Triple dark state structure appears when the rf field and probing field polarizations are perpendicular to each other. Parameters are $\Omega_c = 4\Gamma$, $\Omega_r = \Gamma$.

To study propagation of a weak probing field in the presence of the rf field we first define absorption and refraction functions in an anisotropic medium. The energy dissipation rate in the probing field per unit volume in the atomic medium is defined by the divergence of the energy flux density [26] and is given by

$$Q = \frac{\mathcal{E}^2}{8\pi T} F \tag{17}$$

where the dimensionless function

$$F = 8\pi^2 \Im \left(\chi_x \cos^2\psi + \chi_y \sin^2\psi\right) \tag{18}$$

determines the ratio of the absorbed energy during the period of the oscillation $T = \frac{2\pi}{\omega_p}$ to the free-space energy density of the probing field. In the regions near EIT resonances where the medium can be considered as transparent we may define the group velocity. We neglect the imaginary part of dielectric susceptibilities and define real wave vector $\vec{k}$ which determines the propagation direction. Since magnetic transitions between ground states and the excited state are forbidden the probing magnetic field coincides with the magnetic induction. In this non magnetic case the Maxwell equations are reduced to the Fresnel's ones

$$(k^2 - \frac{\omega_p^2}{c^2})\vec{\mathcal{E}} - (\vec{k}, \vec{\mathcal{E}})\vec{k} = 4\pi\frac{\omega_p^2}{c^2}\vec{\mathcal{P}}. \tag{19}$$

Assuming that the probing light propagates in xOy plane

$$k_x = k\sin\phi, \qquad k_y = k\cos\phi \tag{20}$$

from (19) we obtain dispersion relations

$$k^2 = \frac{\omega_p^2}{c^2}\frac{\epsilon_x^2\cos^2\psi + \epsilon_y^2\sin^2\psi}{\epsilon_x\cos^2\psi + \epsilon_y\sin^2\psi} \tag{21}$$

and

$$\cot\phi\tan\psi = -\frac{\epsilon_x}{\epsilon_y}. \tag{22}$$

Here $\epsilon_a = 1 + 4\pi\chi_a$ is the electric permittitivy. When the dielectric susceptibilities are much smaller than unity we may rewrite (21) in the form

$$k = \frac{\omega_p}{c}(1 + 2\pi\Re\left(\chi_x\cos^2\psi + \chi_y\sin^2\psi\right)). \tag{23}$$

Using (18) and (23) we conclude that absorption and refraction rates are proportional to the imaginary and the real parts of the susceptibility function

$$\chi(\psi) = \chi_x\cos^2\psi + \chi_y\sin^2\psi. \tag{24}$$

An atomic electron subjected to the x -polarized rf field will oscillate in the plane which is transverse to the x axis. Consequenly no atomic dipole moment in x direction can be induced by the rf field. As a result the x component of the susceptibility χ_x will be independent of the rf field in the first order perturbation theory. From (24) we conclude that when the rf and probing fields are polarized in the same direction ($\psi = 0$) the degenerate four level system admits single EIT resonance similar to that of the Lambda system (see Fig 2 (a)). On the other hand when the polarization vectors of the fields are perpendicular to each other the quantum system acquires the triple dark state structure as shown in Fig. 2(b). The shape and position of EIT resonances can be manipulated by adjusting the angle ψ.

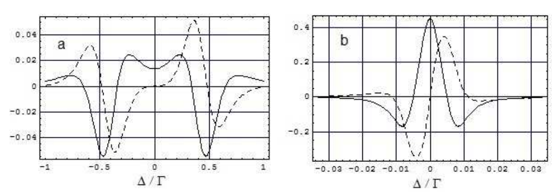

FIG. 3. Imaginary (solid lines) and real (dashed lines) parts of the susceptibility function $\delta\chi$ in the units of λ for the degenerate four level system. (a) Triple dark state structure occurs at $\Omega_c = \Gamma$ and $\Omega_r = 0,1\Gamma$; (b) Double dark state structure occurs at $\Omega_c = 0.1\Gamma$ and $\Omega_r = 0,01\Gamma$

Since the maximum power of the rf (or microwave) field is limited due to experimental difficulties it is of particular interest to examine the influence of a weak rf field on optical properties of a medium. To analyse EIT resonances in this case it is instructive to represent the susceptibility function (24) in the following form

$$\chi(\psi) = \chi(0) + \delta\chi\sin^2\psi \tag{25}$$

where $\chi(0)$ describes absoption and refraction effects in the absence of the rf field (which is the Lambda system, as we have mentioned above) and $\delta\chi$ includes effects induced by the rf field. The contribution of the rf field to absorption and refraction profiles are shown in Fig.3. Depending on the amplitude of the fields, these resonances can have either a double or a triple dark state structure. Fig.3 depicts an important feature of these resonances: The refraction index acquires large values in regions where the absorption function is close to zero or even negative (the region of the optical gain). Such property of the rf field induced resonances can be used to improve the refraction index of the medium.

Imposing the transparency condition $\Im\left(\chi(\psi)\right)=0$ on (25) we obtain the polarization geometry which leads to the emergence of non Raman resonances

$$\sin^2\psi=-\frac{\chi(0)}{\delta\chi}. \tag{26}$$

When the probe field is in the exact resonance with the $E_1\rightarrow E_4$ transition (that is $\triangle=0$) we have

$$\Im\left(\chi(\psi)\right)=\frac{4\lambda\gamma\Gamma^2}{8\gamma\Gamma^2+3\Omega_c^2\Gamma+10\Omega_c^2\gamma} \times\left(\frac{\Gamma\Omega_c^2-2\gamma\Omega_c^2+4\Gamma^2\gamma}{\gamma(\Omega_c^2+2\Gamma\gamma)}\frac{\Omega_r^2}{\Omega_c^2}\sin^2\psi-1\right) \tag{27}$$

Here $\lambda=\frac{\sqrt{2}\varrho d^2}{\hbar\Gamma}$ and we have preserved terms up to the second order in $\frac{\Omega_r}{\Omega_c}$. At $\psi=0$ the imaginary part of the susceptibility function (24) is negative and we have the optical gain for the probing field. The energy of the coupling light is transformed into the one of the probing light by means of the spin-exchange mechanism. The physics of this phenomenon is similar to that of the Hanle effect where the polarization of an incident laser light is changed by a constant magnetic field [30]. In our case the probing field polarization which is directed along the z -axis is transformed into the linear polarization in the xOy plane. By the virtue of (26) and (27) we obtain the polarization configuration

$$\sin^2\psi=\frac{\gamma(\Omega_c^2+2\Gamma\gamma)}{\Gamma\Omega_c^2-2\gamma\Omega_c^2+4\Gamma^2\gamma}\frac{\Omega_c^2}{\Omega_r^2} \tag{28}$$

at which non Raman resonances occur. From (27) we observe that non Raman resonances are critical points at which the probing field absorption turns into the optical gain.

Although an rf (or microwave) source is more readily available and easier to control in comparison with an extra laser field there are practical limitations on the power of the rf field. This creates difficulties in the experimental observations of rf field induced light shifts in Lambda type systems. To observe narrow double dark state resonance (which is the EIT resonance associated with Raman dark states in non-degenerate four level system) one should overcome the Doppler broadening that attenuates the narrow features of optical spectra. Using special Doppler free geometry a narrow double dark state was observed in Rb atomic vapor in [15]. Non Raman resonances are not defined by an rf field induced light shift, they have geometrical origin rather than "energetic". We anticipate that non Raman resonances may provide new possibilities for experimental observations of rf filed (or microwave) effects in multilevel systems. In particular non Raman resonances may be of interest in the metrology science for measuring of an rf field amplitude with high accuracy [31]. The presence of dipole transitions in the optical spectrum makes alkali atoms more suitable for an experimental implementation of the configuration proposed in this work. This can be achieved for example by realization of the degenerate four level system within the Rb D_1 absorption line. The $\mid 1>$, $\mid 1'>$ and $\mid 2>$ states can be set up with three $F=1$ Zeeman sub-levels similarly with the H atom (see the equation (7)), and the remaining $\mid 3>$ state is set up with the $F=2$, $m=0$ Zeeman state. We expect that general properties of EIT resonances for atoms with single outer s electron will resemble the ones which we have established for the atomic hydrogen.

IV. CONCLUSIONS

This article provides a detailed theoretical treatment of EIT resonances in the configuration of Fig.1(a). The degeneracy in the ground state gives rise to the appearance of new type of dark states in the system. Along with two-photon Raman resonances (the triple dark state structure) there are non Raman ones which have geometrical origin. We realize the configuration in the atomic hydrogen and show that non Raman dark states are defined by the geometry of the rf and probing field's polarizations. Using the dressed picture formalism we show that for ultra-cold atoms non Raman resonances are determined by scalar and vector products of the polarization vectors. Properties of Raman and non Raman resonances for a dense hydrogen gas at room temperatures are studied by examing the linear response of atoms to the weak probe field. We take into account the spin-exchage relaxation in the ground state of the atomic hydrogen. Using solutions of the density matrix equations we find that two components of the diagonal susceptibility tensor differ by a factor which is proportional to the magnitude of the rf field. As a result a hydrogen gas becomes anisoptropic for the probing field in the presence of the rf field . Absorption and refraction properties of the medium is defined by the imaginary and the real parts of the susceptibility function (24). The Lambda type EIT resonance appears in the case when the probing field is polarized in the same direction as the rf one. On the other hand when the probing field polarization is perpendicular to that of the rf field the triple dark state structure arises which is the extension of the double dark structure observed in non degenerate four level systems [18, 19]. It is possible to represent optical properties of the medium as the sum of the Lambda type EIT resonance and an rf field induced resonace. The latter is proportional to the

square of the sine function which argument represents the angle between the probing and the rf field's polarizations (see Eq. (25)). We observe an interesting feature of the rf field induced resonance: The refraction index reaches maximal values in regions where the absorption rate is close to zero or even negative (the region of the optical gain). When the probing field is resonant we give the analytic expression for EIT resonances and establish the connection of non Raman dark states with the polarization geometry.

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Appendix A: Linear response

The linear response σ of the atomic system to the weak probe field obeys the Liouville equation

$$\frac{d^{ex}\sigma}{dt} + \frac{d^{sp}\sigma}{dt} = i[H_0, \sigma] + i[V, \rho]. \tag{A1}$$

Here

$$V = -\frac{1}{2}(\Omega_p \mid 4 >< 1 \mid + \Omega_p' \mid 4 >< 1' \mid + H.c) \tag{A2}$$

is the the probe field potential where the Rabi frequencies due to (10) and (14) take the form

$$\Omega_p = \frac{d}{\sqrt{2}\hbar}\mathcal{E}e^{i\psi}, \quad \Omega_p' = \frac{d}{\sqrt{2}\hbar}\mathcal{E}e^{-i\psi}. \tag{A3}$$

Dropping the potential V from (2) we arrive at the Hamiltonian H_0 of "atom + coupling field + rf field" system. The evolution of the atom due to the spontaneous light emission is given by

$$\frac{d^{sp}\sigma_{44}}{dt} = -\Gamma\sigma_{44} \tag{A4a}$$
$$\frac{d^{sp}\sigma_{\beta\beta}}{dt} = \frac{1}{4}\Gamma\sigma_{44} \tag{A4b}$$
$$\frac{d^{sp}\sigma_{4\beta}}{dt} = -\frac{1}{2}\Gamma\sigma_{4\beta}, \tag{A4c}$$

where $\beta = 1, 1', 2, 3$ and Γ is the decay rate of the upper state. Here for simplicity we assume that the ratios of the population decay from the excited state to different ground states are equal. We have used the symbol $\frac{d^{sp}}{dt}$ to distinguish the evolution of σ due to spontaneous emissions (sp) from the evolution of the density matrix due to spin-exchange collisions (ex) which in the linear approximation reads

$$\frac{d^{ex}\sigma_{11}}{dt} = -\frac{\gamma}{2}(\sigma_{11} + \sigma_{1'1'} - \sigma_{22} - \sigma_{33}) \tag{A5a}$$
$$\frac{d^{ex}\sigma_{22}}{dt} = -\frac{\gamma}{2}(3\sigma_{22} - \sigma_{1'1'} - \sigma_{11} - \sigma_{33}) \tag{A5b}$$
$$\frac{d^{ex}\sigma_{1'1'}}{dt} = -\frac{\gamma}{2}(\sigma_{11} + \sigma_{1'1'} - \sigma_{22} - \sigma_{33}) \tag{A5c}$$
$$\frac{d^{ex}\sigma_{33}}{dt} = -\frac{\gamma}{2}(3\sigma_{33} - \sigma_{1'1'} - \sigma_{11} - \sigma_{22}) \tag{A5d}$$
$$\frac{d^{ex}\sigma_{21}}{dt} = -\gamma(\sigma_{21} - \sigma_{1'2}) \tag{A5e}$$
$$\frac{d^{ex}\sigma_{1'2}}{dt} = -\gamma(\sigma_{1'2} - \sigma_{21}) \tag{A5f}$$
$$\frac{d^{ex}\sigma_{1'1}}{dt} = -2\gamma\sigma_{1'1} \tag{A5g}$$
$$\frac{d^{ex}\sigma_{3\alpha}}{dt} = -\gamma\sigma_{3\alpha}. \tag{A5h}$$

Here $\alpha = 1, 1', 2$ and γ is the spin exchange rate. In the general case evolution equations due to spin-exchange collisions contain also quadratic terms in the density matrix components which are proportional to the total spin of an atomic gas [27, 28].

The steady state of an atomic gas prepared by the coupling field and the rf field satisfy the equation

$$\frac{d^{ex}\rho}{dt} + \frac{d^{sp}\rho}{dt} = i[H_0, \rho]. \tag{A6}$$

and is given by the density matrix whose non zero matrix elements are

$$\rho_{11} = \frac{2\gamma(\Gamma^2 + \Omega_c^2) + \Omega_c^2\Gamma}{8\Gamma^2\gamma + 10\Omega_c^2\gamma + 3\Gamma\Omega_c^2} \tag{A7a}$$
$$\rho_{22} = \frac{2\gamma(\Gamma^2 + \Omega_c^2)}{8\Gamma^2\gamma + 10\Omega_c^2\gamma + 3\Gamma\Omega_c^2} \tag{A7b}$$
$$\rho_{44} = \frac{2\gamma\Omega_c^2}{8\Gamma^2\gamma + 10\Omega_c^2\gamma + 3\Gamma\Omega_c^2} \tag{A7c}$$
$$\rho_{42} = i\frac{\Gamma}{\Omega_c}\rho_{44}, \quad \rho_{24} = -i\frac{\Gamma}{\Omega_c}\rho_{44} \tag{A7d}$$

and $\rho_{11} = \rho_{1'1'} = \rho_{33}$.

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